Rankine Heat Pumps' Performance Equations as Temperature-Lift Variables

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Ideal heat pump performance models containing one adjustable parameter and intensive thermodynamic variables were developed by transformation of the energetic functions of Rankine heat pump. Two unknown groups in the formulated model, labeled AKR and AKF, were found to curve fit to two intensive variables as linear functions. The parameters of the linear fits were obtained using the least square method; consequently, AKR and AKF were expressed as temperature-lift variables. Maximum errors of about 5% were encountered at conditions below reduced temperature of 0.8. Better accuracy was obtained in the range of practical temperature lift (i.e., 10–80°C). Only one fit parameter value is required by these models where similar correlations require three or more. Furthermore, unlike some of the available models where different equations are required for each working fluid, these equations are not working fluid specific and do not require thermodynamic properties' tables or charts.
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Introduction

The performance of a heat pump is indicated by its coefficient of performance (COP). As the hypothetical Rankine heat pump cycle approximates ideal vapor compression heat pump,[1–7] the coefficient of performance of Rankine heat pump (COPR) is useful for preliminary design of vapor compression heat pump system and its first law analysis as well as screening of working fluids for the system. As the working fluids' properties are the only considerations in determining the performance of ideal vapor compression heat pump, COPR is given only in terms of the internal thermodynamic properties (enthalpy, entropy, temperature, pressure, etc.) of the system.

The COPR in the heating mode is a quotient of the enthalpy changes accompanying condensation and compression processes in the cycle (see Figure 1). Where enthalpy data are available at the condition of interest, only a direct substitution in the quotient is required. However, this is not usually the case; besides, tabular and graphical data forms are not easily amenable to computer-aided calculations. So, there is always the need to use some equations/correlations to evaluate the coefficient. A fundamental property relation gives enthalpy as a function of entropy, volume, temperature, and pressure,[1] but correlations of entropy and volume in term of temperature and pressure can be difficult to use due to data availability. This observation justifies the need for COPR models, which are expressed in terms of intensive variables such as temperature and pressure—the process operating variables that are normally specified. However, the available models of this kind are empirical correlations, which can neither be readily extrapolated with confidence nor applied as generalized models.[3,8–10] Furthermore, the available software programs usually use correlations requiring three or more parameters to evaluate enthalpy and other functions that may be required to evaluate COP. A form of

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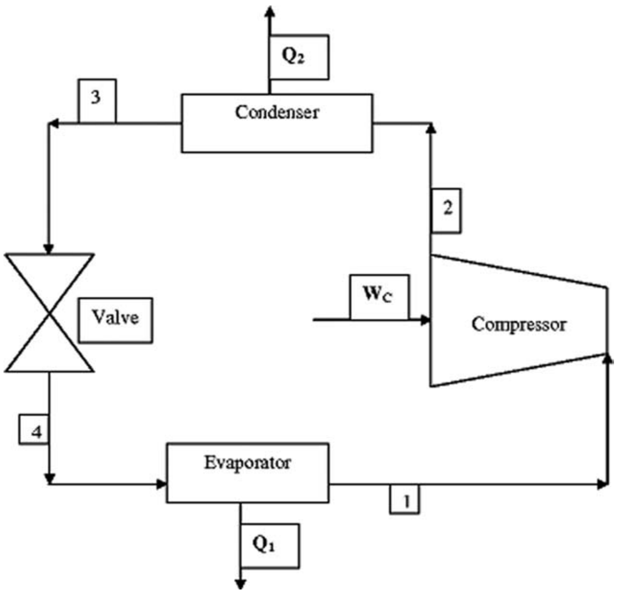

Figure 1. Vapor compression heat pump cycle.

such equations is given by Patwardhan and Patwardhan[11] in which the embedded correlations for specific heat and latent heat of vaporization of the working fluids contain more than three fit parameters. Much as COP models which are based on intensive variables are desirable because limited data are required for their use, reported works on them in the literature are scanty.

Even then generalized mechanistic COP models which are based on intensive variables such as temperature and or pressure could be obtained using semiempirical approach. The fundamental property relations of enthalpy changes for isentropic compression and isobaric condensation processes could be transformed using Maxwell equations and other partial derivatives identities,[2,12] Bridgman tables,[13,14] and techniques for interconversion of partial derivatives[14–16] to give COPR largely in terms of temperature and pressure. Thereafter, the non-temperature/pressure terms in the formulated model, if not constant, could be correlated to temperature/pressure variables as linear, power or logarithmic function fit, among others.[9] Therefore, such models would contain a maximum of two fit parameters.

In this work, generalized models, based on intensive variables, were formulated for COP of ideal vapor compression heat pumps. The formulated models were thereafter calibrated by curve fitting two unknown groups (AKR and AKF) in the models to temperature-lift variables and determining the parameters of the fits. The accuracy and limit of validity of the resulting models were thereafter established for 18 working fluids. Furthermore, expressions for evaluating heating duty at low to moderate pressure were derived.

Procedure

The methods used for the formulation of the models, their calibration, and subsequent validation are described subsequently.

Formulation of the model

Transformation of the Enthalpy Ratio. The COPR is the enthalpy change ratio of condensation and compression processes in vapor compression heat pump. That is:

$$\mathrm{COPR} = \frac{(\Delta H)_\mathrm{P}}{(\Delta H)_\mathrm{S}} \tag{1}$$

where $(\Delta H)_\mathrm{P}$ is isobaric enthalpy change on condensation, and $(\Delta H)_\mathrm{S}$ is isentropic enthalpy change on compression.

The differential fundamental property relation of enthalpy (closed system) is given as:

$$dH = TdS + VdP \tag{2}$$

For simple substances and azeotropes, condensation and evaporation could be represented as (from Eq. 2):

$$(dH)_\mathrm{T,P} = (TdS)_\mathrm{T,P} \tag{3a}$$

and for compression process we have:

$$(dH)_\mathrm{S} = (VdP)_\mathrm{S} \tag{3b}$$

or in integral forms as:

$$(\Delta H)_\mathrm{P} = \int (TdS)_\mathrm{P} \tag{4a}$$

$$(\Delta H)_\mathrm{S} = \int (VdP)_\mathrm{S} \tag{4b}$$

Substituting Eqs. 4a and 4b in Eq. 1 gives:

$$\mathrm{COPR} = \int \frac{(TdS)_\mathrm{P}}{(VdP)_\mathrm{S}} \tag{5}$$

Noting that $\left(\frac{\partial T}{\partial P}\right)_\mathrm{S} = \left(\frac{\partial V}{\partial S}\right)_\mathrm{P}$ (a Maxwell equation), and multiplying the right-hand side of Eq. 5 by $\left(\frac{\partial P}{\partial T}\right)_\mathrm{S}\left(\frac{\partial V}{\partial S}\right)_\mathrm{P}$ gives:

$$\mathrm{COPR} = \int \frac{(TdV)_\mathrm{P}}{(VdT)_\mathrm{S}} \tag{6}$$

Formulated Model. The assumptions and approximations made in the formulation of the model are (a) phase transition occurs at constant temperature (i.e., no superheat) and (b) volume change with condensation approximates saturated vapor volume (i.e., $V_\mathrm{g} - V_\mathrm{l} = V_\mathrm{g}$)

Assumptions (a) and (b), respectively, translate to Eqs. 7 and 8.

$$\mathrm{COPR} = T \int \frac{(dV)_\mathrm{P}}{(VdT)_\mathrm{S}} \tag{7}$$

$$\int (dV)_\mathrm{P} \cong V_\mathrm{g} \cong zRT/P \tag{8}$$

where V_g is saturated vapor specific or molar volume, V_l is saturated liquid specific or molar volume, T is temperature, P is pressure, z is compressibility factor, and R is gas constant.

now,

$$V_{\mathrm{S}} = \left(\frac{\partial H}{\partial P}\right)_{\mathrm{S}}, \left(\frac{\partial H}{\partial P}\right)_{\mathrm{S}} = -\left(\frac{\partial H}{\partial S}\right)_{\mathrm{P}}\left(\frac{\partial S}{\partial P}\right)_{\mathrm{H}} \text{ and } T = \left(\frac{\partial H}{\partial S}\right)_{\mathrm{P}}$$
$$\Rightarrow V_{\mathrm{S}} = -T\left(\tfrac{\partial S}{\partial P}\right)_{\mathrm{H}}$$

Isenthalpic curves in P–S chart for pure substances are fairly linear[16] and so a typical slope $(\partial S/\partial P)_{\mathrm{H}}$ could be taken as a constant, k_{H}. Thus, the integral form of the denominator of Eq. 7 is written as:

$$\int (VdT)_{\mathrm{S}} = -(\partial H/\partial P)_{\mathrm{H}} \cdot \int (TdT)_{\mathrm{S}}$$
$$= k_{\mathrm{H}} \int_{T_{\mathrm{EV}}}^{T_{\mathrm{CO}}} TdT = 0.5k_{\mathrm{H}}(T_{\mathrm{CO}}^2 - T_{\mathrm{EV}}^2) \quad (9)$$

Substituting in Eq. 7 with Eqs. 8 and 9 gives:

$$\mathrm{COPR} = \left(\frac{2RZ_{\mathrm{CO}}}{k_{\mathrm{H}}P_{\mathrm{CO}}}\frac{T_{\mathrm{CO}}}{(T_{\mathrm{CO}}+T_{\mathrm{EV}})}\frac{T_{\mathrm{CO}}}{T_{\mathrm{CO}}-T_{\mathrm{EV}}}\right) \quad (10)$$

Equation 10 is the formulated model obtained through transformation of the enthalpy ratio.

Calibration of the models

This step involves changing the non-intensive unknown terms in Eq. 10 to intensive terms by simple curve fitting and parameter estimation of the fits. From preliminary evaluations, two groups made up of the unknowns in Eq. 10 were identified to be suitable for curve fitting to temperature–pressure variables. The groups are:

- $RZ_{\mathrm{CO}}/k_{\mathrm{H}}P_{\mathrm{CO}}$, which is a dimensionless group, labeled AKR.
- $Z_{\mathrm{CO}}/k_{\mathrm{H}}$, which is a group of two unknown quantities, labeled AKF.

Consequent on this, Eq. 10 becomes:

$$\mathrm{COPR} = \left(2\mathrm{AKR}\frac{T_{\mathrm{CO}}}{(T_{\mathrm{CO}}+T_{\mathrm{EV}})}\frac{T_{\mathrm{CO}}}{T_{\mathrm{CO}}-T_{\mathrm{EV}}}\right) \quad (10a)$$

$$\mathrm{COPR} = \left(\frac{2R.\mathrm{AKF}}{P_{\mathrm{CO}}}\frac{T_{\mathrm{CO}}}{(T_{\mathrm{CO}}+T_{\mathrm{EV}})}\frac{T_{\mathrm{CO}}}{T_{\mathrm{CO}}-T_{\mathrm{EV}}}\right) \quad (10b)$$

Numerical values for AKR and AKF, COPR, and some selected variables of temperature and or pressure were computed at different evaporating and condensing temperatures with Fortran program. COPR was calculated with Eq. 11a after substituting H_{D1} with Eqs. 11b and 11c for dry and wet working fluids, respectively. The expressions for evaluating AKR and AKF are given by Eqs. 12a and 12b, respectively.

$$\mathrm{COPR} = \frac{H_{\mathrm{D1}} - H_{\mathrm{D3}}}{H_{\mathrm{D1}} - H_{\mathrm{S2}}} \quad (11a)$$

$$H_{\mathrm{D1}} = H_{\mathrm{D2}} + (\theta_{\mathrm{S2}} - \theta_{\mathrm{D2}})T_{\mathrm{CO}} \quad (11b)$$

$$H_{\mathrm{D1}} = H_{\mathrm{D3}}X + H_{\mathrm{D2}}(1 - X) \quad (11c)$$

$$\mathrm{AKR} = \frac{0.5[\mathrm{COPR}.(T_{\mathrm{CO}}^2 - T_{\mathrm{EV}}^2)]}{T_{\mathrm{CO}}^2} \quad (12a)$$

$$\mathrm{AKF} = \frac{0.5[\mathrm{COPR}(T_{\mathrm{CO}}^2 - T_{\mathrm{EV}}^2)P_{\mathrm{CO}}]}{R.T_{\mathrm{CO}}^2} \quad (12b)$$

where $\theta_{\mathrm{D1}} = \theta_{\mathrm{S2}} = \theta_{\mathrm{D3}}X + \theta_{\mathrm{D2}}\,(1 - X)$; X is the liquid fraction of the wet vapor at final state of isentropic compression, D1, H and θ are, respectively, the mass (specific) enthalpy and entropy of the fluid at the points indicated by the subscripts, subscripts D3 and D2 are the saturation points on liquid and vapor lines, respectively, subscript D1 is in superheated and wet regions for dry and wet fluids, respectively, D1, D2, and D3 are points at the heat sink (isobaric condensing line), and S2 stands for saturation vapor point at the heat source (isobaric evaporating line).

The computations were made for selected 17 working fluids, over the range of conditions from the triple point to critical point, using thermodynamic property tables as data sources.[17,18]

The temperature and or pressure variables to which AKR and AKF were fitted included temperature lift, compression ratio, temperature ratio, and some temperature-lift variables such as $(T_{\mathrm{CO}} - T_{\mathrm{EV}})T_{\mathrm{CO}}/T_{\mathrm{EV}}$ and $(T_{\mathrm{CO}} - T_{\mathrm{EV}})T_{\mathrm{EV}}$. Other temperature-lift variables were also investigated by Olawale.[19] The curve fittings were carried out with Microsoft Excel 2003. Estimation of the fit parameters was done by least square method. Results are given in Figures 2 and 3 and Table 1.

Validation of models

AKR and AKF fitted linearly to two temperature-lift variables within the range of theoretically feasible operating conditions of vapor compression heat pump. The two variables were substituted in Eqs. 10a and 10b, with the appropriate parameter values as given in Table 1. Computations were then carried out to assess the accuracy and validity of the models as given in Table 2. Thermodynamic data were used to calculate actual COPR and these were then compared with COPR obtained using the models developed.

Evaluation of heating duty

Heating and cooling duties of heat pump are related to COP as the coefficient is the quotient of the heating duty (in heating mode) or cooling duty (in cooling mode) and the compression power $(\dot{W}_{\mathrm{C}})$ or its equivalents. Thus, heating duty $(\dot{H}_{\mathrm{D}})$ for Rankine heat pump could be given as:

$$\dot{H}_{\mathrm{D}} = \mathrm{COPR} \cdot \dot{W}_{\mathrm{C}} \quad (13a)$$

For ideal vapor compression (Rankine) heat pump, work of compression is carried out isentropically, and at low to moderate pressure compression power is given as[1]:

$$\dot{W}_{\mathrm{C}} = R(T_{\mathrm{CO}} - T_{\mathrm{EV}})/(\gamma - 1) \quad (13b)$$

where $\gamma = C_{\mathrm{p}}/C_{\mathrm{v}}$ (C_{p} and C_{v} are specific heat capacities at constant pressure and volume, respectively).

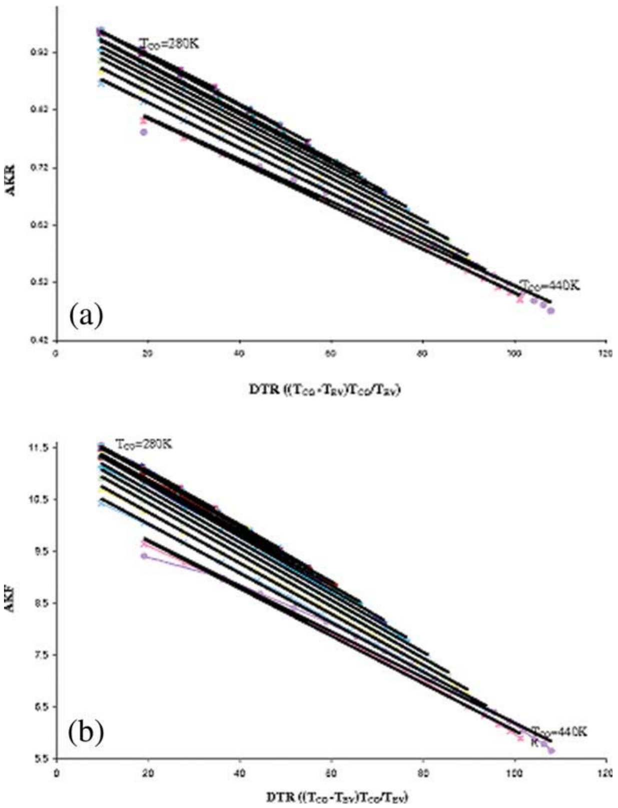

Figure 2. **(a) Condensing isotherms of AKR fitted to DTR for R-21. (b) Condensing isotherms of AKF fitted to DTR for R-21.**

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Results and Discussion

Calibration of the models

In the feasible operating range (triple to critical point), AKR and AKF did not fit linearly to temperature lift, temperature ratio, and compression ratio but rather to temperature-lift variables such as $(T_{CO} - T_{EV})T_{CO}/T_{EV}$ and $(T_{CO} - T_{EV})T_{EV}$.

The intercepts for AKR and AKF linear fit of the two temperature-lift variables tended, respectively, to 1.0 and 12.0 in all cases as can be seen from Figures 2a, b and 3a, b. Similar observations were recorded for all other working fluids investigated. The drop in the intercepts' values from those quoted above in the critical point neighborhood might be due to the usual breakdown of thermodynamic relations in this region.[1] The forms of the fits are thus: ($R^2 \geq 0.99$; adj $R^2 \geq 0.90$)

$$\text{AKR} = a_{R1}(T_{CO} - T_{EV})T_{EV} + 1 \qquad (14a)$$

$$\text{AKR} = a_{R2}(T_{CO} - T_{EV})^{T_{CO}}\!/_{T_{EV}} + 1 \qquad (14b)$$

$$\text{AKF} = a_{F1}(T_{CO} - T_{EV})T_{EV} + 12 \qquad (14c)$$

$$\text{AKF} = a_{F2}(T_{CO} - T_{EV})^{T_{CO}}\!/_{T_{EV}} + 12 \qquad (14d)$$

On substituting for AKR and AKF with Eqs. 14a–14d in Eqs. 10a and 10b, as appropriate we have Eqs. 15a and 15b (for AKR fit) and Eqs. 16a and 16b (for AKF) as the final forms of the models developed.

$$\text{COPR} = 2.0[a_{R1}(T_{CO} - T_{EV})T_{EV} + 1] \times \left(\frac{T_{CO}}{T_{CO} + T_{EV}}\right)\left(\frac{T_{CO}}{T_{CO} - T_{EV}}\right) \qquad (15a)$$

$$\text{COPR} = 2.0\left[a_{R2}(T_{CO} - T_{EV})\left(\frac{T_{CO}}{T_{EV}}\right) + 1\right] \times \left(\frac{T_{CO}}{T_{CO} + T_{EV}}\right)\left(\frac{T_{CO}}{T_{CO} - T_{EV}}\right) \qquad (15b)$$

$$\text{COPR} = \frac{2.R}{P_{CO}}[a_{F1}(T_{CO} - T_{EV})T_{EV} + 12] \times \left(\frac{T_{CO}}{T_{CO} + T_{EV}}\right)\left(\frac{T_{CO}}{T_{CO} - T_{EV}}\right) \qquad (16a)$$

$$\text{COPR} = \frac{2.R}{P_{CO}}\left[a_{F2}(T_{CO} - T_{EV})\left(\frac{T_{CO}}{T_{EV}}\right) + 12\right] \times \left(\frac{T_{CO}}{T_{CO} + T_{EV}}\right)\left(\frac{T_{CO}}{T_{CO} - T_{EV}}\right) \qquad (16b)$$

where R is the gas constant for the fluid of interest, (kJ/kg. K), and a_{R1-F2} are the slopes of the linear plots fit parameters.

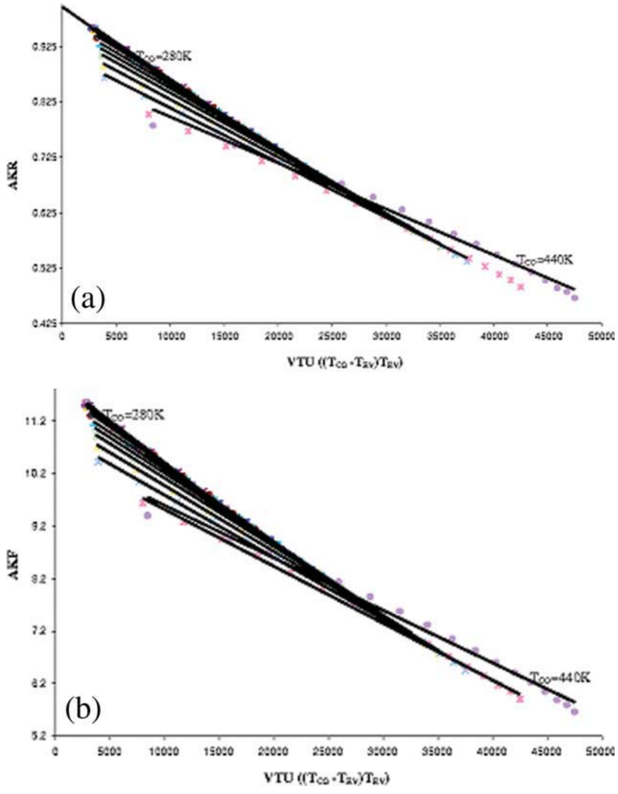

Figure 3. **(a) Condensing isotherms of AKR fitted to VTU for R-21. (b) Condensing isotherms of AKF fitted to VTU for R-21.**

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Table 1. Parameters of AKR and AKF Fits to Temperature-Lift Variables

Working Fluids		Values of Fit Parameters for Use in Eqs. 15–18			
Formula	Code*	a_{R1}	a_{R2}	a_{F1}	a_{F2}
CCl_3F	R-11	−2E-05	−0.0049	−0.0002	0.058
$CClF_3$	R-13	−2E-05	−0.0066	−0.0004	0.0670
$CHCl_2F$	R-21	−1E-05	−0.0040	−0.0002	0.0484
$CHClF_2$	R-22	−2E-05	−0.0053	−0.0002	0.0636
$C_2H_3ClF_2$	R-142b	−2E-05	−0.0044		
CHF_3	R-23	−3E-05	−0.0054	−0.0003	0.0650
CH_2F_2	R-32	−2E-05	−0.0045	−0.0002	0.0543
$C_2H_4F_2$	R-152a	−2E-05	−0.0047		
C_3H_8	R-290	−2E-05	−0.0060		
NH_3	R-717	−2E-05	−0.0041		
CF_3Br	R-13b1	−3E-05	−0.0055	−0.0003	0.0679
$C_2Cl_3F_3$	R-113	−2E-05	−0.0058	−0.0002	0.0581
$C_2Cl_2F_4$	R-114	−2E-05	−0.0053		
$C_2HCl_2F_3$	R-123	−2E-05	−0.0052		
C_2HClF_4	R-124	−2E-05	−0.0053		
C_2HF_5	R-125	−2E-05	−0.0063		
iso-C_4H_{10}	R-600a	−2E-05	−0.0062		
C_4H_{10}	R-600	−2E-05	−0.0057		

*ASHRAE Code.

The values of the fit parameters for 17 working fluids are given in Table 1. These values could be observed to lie within narrow band.

Expressions for heating duty

Models for heating duty were obtained by substituting Eq. 13b and COPR models (Eqs. 15 and 16) in Eq. 13a to give Eqs. 17 and 18.

$$\dot{H}_{\mathrm{D}} = \frac{2R}{\gamma - 1}[a_{\mathrm{R1}}(T_{\mathrm{CO}} - T_{\mathrm{EV}})T_{\mathrm{EV}} + 1]\frac{T_{\mathrm{CO}}^2}{T_{\mathrm{CO}} + T_{\mathrm{EV}}} \tag{17a}$$

$$\dot{H}_{\mathrm{D}} = \frac{2R}{\gamma - 1}\left[a_{\mathrm{R2}}(T_{\mathrm{CO}} - T_{\mathrm{EV}})\frac{T_{\mathrm{CO}}}{T_{\mathrm{EV}}} + 1\right]\frac{T_{\mathrm{CO}}^2}{T_{\mathrm{CO}} + T_{\mathrm{EV}}} \tag{17b}$$

$$\dot{H}_{\mathrm{D}} = \frac{2R^2}{\gamma - 1}[a_{\mathrm{F1}}(T_{\mathrm{CO}} - T_{\mathrm{EV}})T_{\mathrm{EV}} + 12]\frac{T_{\mathrm{CO}}^2}{T_{\mathrm{CO}} + T_{\mathrm{EV}}} \tag{18a}$$

$$\dot{H}_{\mathrm{D}} = \frac{2R^2}{\gamma - 1}\left[a_{\mathrm{F2}}(T_{\mathrm{CO}} - T_{\mathrm{EV}})\frac{T_{\mathrm{CO}}}{T_{\mathrm{EV}}} + 12\right]\frac{T_{\mathrm{CO}}^2}{T_{\mathrm{CO}} + T_{\mathrm{EV}}} \tag{18b}$$

In Eqs. 17 and 18, γ could be taken as 1.3.[1] It could be seen that each of the expressions for heat duty given by Eqs. 17 and 18 require only the fit parameter (i.e., a_{R1}, a_{R2}, a_{F1}, or a_{F2}) just like COPR models.

Validation of models

From the results presented in Table 2, it is seen that the models gave satisfactory results for conditions below reduced condensing temperature (T_r) of 0.8. However, higher error values were obtained at high condensing temperature. One obvious reason for this is that values of intercept of the AKR linear fits at condensing temperature close to the critical point are less than the chosen value of 1.0. In addition, it is known that thermodynamic relations usually break down in the vicinity of critical region,[1] and as the equations were derived using thermodynamic relations, the observation could be justified.

Table 2. Maximum Error Percent of the Models for Different Working Fluids

Working Fluids		Maximum Error, % (with T_r)			
Formula	Code*	Equation 15a		Equation 15b	
CCl_3F	R-11	7.13	(0.61)	6.63	(0.81)
$CClF_3$	R-13	5.23	(0.79)	5.35	(0.86)
$CHCl_2F$	R-21	6.06	(0.66)	5.68	(0.77)
$CHClF_2$	R-22	5.75	(0.78)	6.20	(0.84)
CHF_3	R-23	6.28	(0.77)	3.89	(0.84)
CH_2F_2	R-32	5.88	(0.76)	5.50	(0.85)
$C_2H_3F_3$	R-142b	6.82	(0.68)	5.56	(0.73)
$C_2H_4F_2$	R-152a	6.43	(0.73)	6.28	(0.80)
C_3H_8	R-290	6.81	(0.45)	5.26	(0.76)
NH_3	R-717	6.22	(0.64)	5.68	(0.86)
H_2OCF_3Br	R-13b1	7.01	(0.68)	6.71	(0.79)
$C_2Cl_3F_3$	R-113	7.34	(0.78)	6.89	(0.63)
$C_2Cl_2F_4$	R-114	5.28	(0.79)	5.52	(0.79)
$C_2HCl_2F_3$	R-123	8.46	(0.68)	5.33	(0.79)
C_2HClF_4	R-124	6.74	(0.87)	5.89	(0.79)
C_2HF_5	R-125	6.07	(0.77)	6.31	(0.86)
C_4H_{10}	R-600	6.21	(.52)	5.78	(0.73)

*ASHRAE Code.

Despite the observed low accuracy at reduced temperature above 0.8, the model has significant practical use for vapor compression heat pump system. Table 2 showed that maximum errors up to 8 and 7% were, respectively, obtained with Eqs. 15a and 15b at conditions below 0.8 T_r. Indeed, within temperature lift range of 10–80°C, 0–5% error range is typical for the working fluids investigated. This is particularly true for Eq. 15b where $(T_{CO} - T_{EV})T_{CO}/T_{EV}$ is used as the linear fit of AKR (see Table 3). This error level is good enough for engineering problems[10,20]; more so for heat pump applications, the region of useful applications fall within this range. In a related work, Patwardhan and Patwardhan[11] reported 3–4% error with COPR model that

Table 3. Error Range at Temperature Lift Between 10 and 80°C up to $T_r \approx 0.8$

Working Fluids		Absolute Error Range, %	
Formula	Code*	Equation 15a	Equation 15b
CCl_3F	R-11	1.03–>10	0.17–2.62
$CClF_3$	R-13	0.40–5.23	0.01–2.68
$CHCl_2F$	R-21	1.42–9.41	0.00–6.43
$CHClF_2$	R-22	0.00–5.35	0.10–3.81
CHF_3	R-23	0.27–8.52	0.02–2.71
CH_2F_2	R-32	0.01–5.80	0.25–2.84
$C_2H_3F_3$	R-142b	0.16–>10	0.02–5.06
$C_2H_4F_2$	R-152a	0.03–>10	0.01–2.79
C_3H_8	R-290	0.03–>10	0.01–4.11
NH_3	R-717	0.12–>10	0.09–3.16
H_2OCF_3Br	R-13b1	0.38–>10	0.02–4.40
$C_2Cl_3F_3$	R-113	0.14–>10	0.11–3.58
$C_2Cl_2F_4$	R-114	0.16–5.65	0.10–3.43
$C_2HCl_2F_3$	R-123	0.18–>10	0.09–4.06
C_2HClF_4	R-124	0.01–2.98	0.13–5.89
C_2HF_5	R-125	0.34–7.80	0.13–2.98
C_4H_{10}	R-600	0.02–8.13	0.09–2.75

T_r is the reduced condensing temperature.
*ASHRAE Code.

incorporated specific heat capacity and heat of evaporation. Even though the models in this work contain only one parameter, the lower limit of error is less than the one reported by Patwardhan and Patwardhan.[11]

The error values appeared less sensitive to temperature lift with Eq. 15b than Eq. 15a. The reason for this is not immediately obvious but might not be unconnected with the presence of evaporating temperature to the power of two in VTU $[(T_{CO} - T_{EV})T_{EV}]$ when compared with one in DTR $[(T_{CO} - T_{EV})T_{CO}/T_{EV}$. The accuracy of the models did not appear to be related to the chemical or thermodynamic properties of the working fluids.

At low to moderate pressure, the derived expressions for heat duty of the system would give accuracy level comparable to those obtained for COPR. These models could be extended to real systems if factors accounting for irreversibility during heat rejection (due to nonisothermal condensation, temperature gradient, and nonequilibrium heat transfer) and energy addition (due to nonisentropic compression and mechanical inefficiency) are incorporated.

Conclusions

Models for evaluating COP for ideal vapor compression (Rankine) heat pump that require no thermodynamic data have been developed. Unlike the available models of similar forms, these models are generalized in nature in that the same equation is applicable to all fluids and they contain only one fit parameter. The best model gave error less than 5% for the 17 working fluids within the range of useful operating conditions for heat pump (i.e., at 10–80°C temperature lift and below reduced condensing temperature of 0.8). The models can be extended to practical system by including factors that account for irreversibilities at the condenser and compressor. The models developed are useful for computer-aided preliminary design and first law analysis as well as fast manual computations for heat pump system. The models can as well be used for thermodynamic assessment of potential fluids as substitutes for CFCs and HCFCs even when thermodynamic properties are lacking. Analogous models of heating duty were derived for ideal vapor compression heat pump.

Notation

a_{F1}, a_{F2}, a_{R1}, a_{R2} = parameters (slopes) of linear fits
b_1, b_2 = parameters (intercepts) of linear fits
CFCs = chlorofluorocarbons
COP = coefficient of performance
COPR = Rankine coefficient of performance
C_p = specific heat capacities at constant pressure, kJ/kg
C_v = specific heat capacities at constant volume, kJ/kg
H = specific enthalpy, kJ/kg
HCFCs = hydrochlorofluorocarbons
$\dot{H}_D$ = heating duty, kJ/s
H_{D1} = specific enthalpy at superheated and wet region, kJ/kg
H_{D1} = specific enthalpy at superheated and wet region, kJ/kg
H_{D2} = specific enthalpy of saturated liquid at isobaric condensation, kJ/kg
H_{D3} = specific enthalpy of saturated vapor at isobaric condensation, kJ/kg
HFC = hydrofluorocarbon working fluids
H_{S2} = specific enthalpy of saturated vapor at isobaric vaporization, kJ/kg
k_H = $(\partial S/\partial P)_H$
P = Pressure, bar or kPa
P_{CO} = condensing pressure, bar or kPa
P_{EV} = evaporating pressure, bar or kPa
R = universal gas constant, kJ/(kmol °C)
S = specific entropy, kJ/(kg K)
T = Temperature, °C or K
T_{CO} = condensing temperature, °C or K
T_{EV} = evaporating temperature, °C or K
W_C = work of compression, kJ/kg
$\dot{W}_C$ = compression power, kJ/s
Z_{CO} = pseudocompressibility factor at condensing condition

Greek letters

∂ = partial differential
γ = specific heat ratio (C_p/C_v)
θ = specific entropy, kJ/(kg K)
θ_{D1} = specific entropy of wet vapor at isobaric condensation, kJ/kg
θ_{D2} = specific entropy of saturated liquid at isobaric condensation, kJ/kg
θ_{D3} = specific entropy of saturated vapor at isobaric condensation, kJ/kg
θ_{S2} = specific entropy of saturated vapor at isobaric vaporization, kJ/kg

Acronyms

AKF = unknown group Z_{CO}/k_H
AKR = dimensionless group $RZ_{CO}/k_H P_{CO}$
DTR = temperature-lift variable given as $(T_{CO} - T_{EV})T_{CO}/T_{EV}$
VTU = temperature-lift variable given as $(T_{CO} - T_{EV})T_{EV}$

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